Overview Annexes A “Failure criterion of wood”

Introduction

This publication is part of compilation of work of the author to a total rigorous theory, containing the latest developments with goal of a thesis and book. The appended articles are given in full as acknowledgment for the original journal publication.
The developed exact theory is given in the appended 4 publications denoted by “A”, thus: vdPut A(1982), A(1993), A(2005) and A(2009). Other important derivations and applications are mentioned in these 3 publications. The theory in all appended publications was derived by T.A.C.M. van der Put, denoted by: vdPut as reference.
In vdPut A(1982), the complete theory of the failure criterion is given, based on clear wood data. In vdPut A(1993), a further discussion and theoretical extension is given, based on data of timber, (wood with defects), focused on the essential part for design, and design rules for Eurocode 5. Although this last was accepted by CIB-W18 and the Eurocode 5 Committee, it never was implemented and also the old Norris failure criterion disappeared from Eurocode 5, and because also all other exact calculation methods are continuously replaced by empirical, thus unreliable, nonsense (see e.g. vdPut D(2012a)), the Eurocode 5 now has no meaning any more for structural design with calculable reliability and reflects the total ignorance of the, by censorship excluded exact theory of the last decades of CIB-W18.
It is always possible and necessary to apply exact theory, as only possibility to guarantee a right calculable reliability. Some day there will be a necessary revival of exact design, when Society does no longer, accept the collapse of buildings with e.g. 1138 dead and thousands of wounded as in Dhaka. At this moment, nearly a year after the collapse, are 38 teams controlling still 1500 clothing ateliers and register everywhere visible overloading by far too thin columns and beams at the wrong places. This confirms the fact that collapse certainly can not be caused by inferior material and overloading, but only is possible by many, on all levels, interrelated fundamental faults of design, due to total absence of knowledge of exact theory by the current generation. Probably, the denial of responsibility for this, leads to denial of the possibility of exact theory as determining law of nature. But, as scientist, it is a duty to base this opinion on study of e.g. the here given theory and on discussion with the author. For my last, about 50, theory publications there never was a reviewer who discussed the content and did show to know what was presented and what theory necessarily was involved.

A.1. Discussion of annexes A about the exact failure criterion of wood

The exact yield or failure criterion has to be applied to make a real prediction of strength possible in all circumstances. By vdPut A(1982) was for the first time shown that part of the Tsai-Wu criterion eq.(A-1) applies to wood and may represent the exact failure criterion. This tensor-polynomial Tsai-Wu equation was shown to act as a polynomial expansion of the real failure surface. The polynomial basis, discussed in section 2.6 of A(1982), appeared to be not transverse isotropic, as could be expected from the layered structure and isotropic matrix, but appeared to be orthotropic, in accordance with the plane stress processes of fracture and flow at the weakest layers. For tensile failure, this evidently is due to flat initial cracks in the main material planes, because only the stresses in one plane are magnified by such a flat crack. For compression this is due to microscopic kinks and creases in the cell walls, leading to shear planes (called creases, shear lines or slip-lines), thus to movement in a plane. This property is applied as basic feature for new fracture mechanics- and local compression strength derivations in the appended literature: e.g. C(2011) and D(2008a).
In A(1982), based on clear wood data, all aspects and all possible transformations are discussed completely. The tensor polynomial form:

$F_i\sigma_i + F_{ij}\sigma_i\sigma_j + F_{ijk}\sigma_i\sigma_j\sigma_k + ... = 1$, (A-1)
regarded as expansion of the real failure criterion shows that not all terms are needed, because data fitting confirmed the following symmetry conditions. The principal directions of strength can be regarded to be orthogonal and therefore the odd higher order terms, F_{ijk}, should be omitted and the failure surface for wood thus reduces to:

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \qquad \text{(A-2)}$$

For reasons of energetic reciprocity $F_{ij} = F_{ji} \quad (i \neq j)$ and because wood also shows to be orthotropic in the main planes, the interaction between the shear stresses can be disregarded $F_{ij} = 0 \quad (i \neq j;\, i, j = 4,5,6)$ and because for the same reasons of orthotropic symmetry in the main material planes, (axial - tangential – radial) the shear strength has to be, and is, identical in positive and negative direction, the odd-order terms of σ_6 are zero and such coupling between normal- and shear strength vanish: $F_6 = F_{16} = F_{26} = 0$, and eq.(A-2) becomes:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 = 1 \qquad \text{(A-3)}$$

For a thermodynamic real surface (i.e. positive strain energy) the values of: F_{ii} must be positive and also the failure surface cannot be open-ended. This gives the restriction: $F_{ii}F_{jj} > F_{ij}^2$ (no summation convention), thus in eq.(A-3): $F_{11}F_{22} - F_{12}^2 > 0$.
($F_{11}F_{22} - F_{12}^2 < 0$ = hyperboloid and $F_{11}F_{22} - F_{12}^2 = 0$ is parabolic; both open ended curves).
It was shown, for the first time, in A(1982), that the second-degree tensor-polynomial represents initial “flow,” of wood, which represents the orthotropic extension of the isotropic critical distortional energy criterion, providing the necessary basis for exact solutions according to limit analysis. Because an isotropic matrix of a material may sustain large hydrostatic pressures without yielding, yield depends on a critical value of the distortional energy. The exact derivation of this principle is given in A(2009) based on linear mapping. In A(1982), section 2.1.2, this derivation was based on the application of pre-stress in such a way that isotropic symmetry occurs in the resultant effective stresses, making the use of the isotropic critical distortional energy equation possible. At the so obtained energy, the energy of the pre-stress was subtracted to obtain the real critical distortional energy equation.
It appeared empirically, that the transverse strengths, (compression, tension and shear in any direction) with e.g. the peculiar behavior of the off-axis compression strengths with a minimal strength at 60^0, given by Fig.6 of A(1982), are precisely described by the second order polynomial, without need of higher order terms. By Fig. 9 of A(1982), (given below) is shown that after initial flow in compression, hardening occurs and after some equal amount of plastic strain at hardening (in all directions), the difference between the off-axes strengths has disappeared and the strength behavior is isotropic. At this point, where empty spaces in

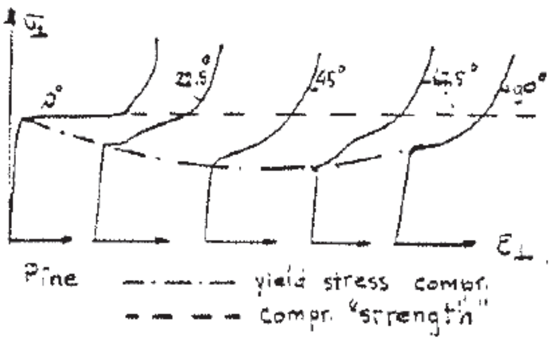


Fig 9 of A(1982). Hardening to the isotropic state in transverse direction wood are pressed away, the isotropic strength of the isotropic matrix becomes determining. At the failure state, wood thus can be regarded as an isotropic material, reinforced in axial direction. This is applied in C(2011) for the exact derivation of the Wu- mixed mode fracture equation. Initial flow, according to the orthotropic second order polynomial, also applies for the longitudinal strengths what leads to failure when the test or test setup becomes instable at this initial flow. This applies e.g. for the off-axis tensile tests. When the test remains stable after initial flow, hardening is possible. The example, discussed in A(1982), shows hardly hardening in the tangential plane of the oblique-grain compression test, project B, while project A, in the radial plane, shows an increasing shear strength with increasing compression stress normal to this plane (see fig. 10 of A(1982), below). For that case is, as polynomial description, the next coupling term between σ_2 and σ_6 needed (what appeared to be sufficient) and because the shear strength in the main planes is independent of the sign of the stress, odd terms in index 6 (p.e. F_6, F_{16}, F_{26}) have to disappear and a higher order term F_{266} is needed, and the failure surface thus becomes for the plane fracture mechanics problem $\sigma_1 = 0$, in the radial plane:

$$F_2\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 3F_{266}\sigma_2\sigma_6^2 = 1 \qquad \text{(A-4)}$$

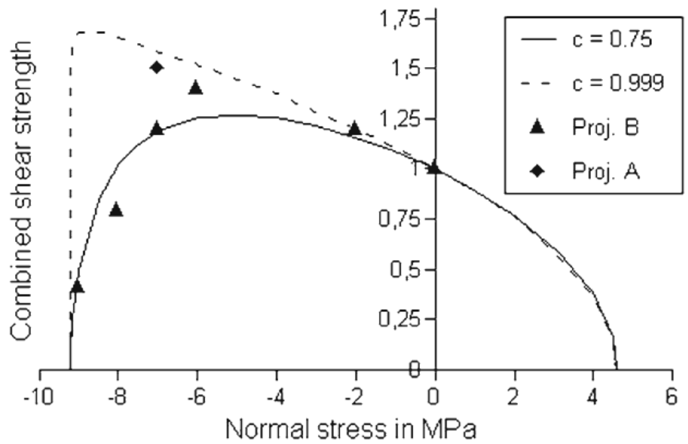


Figure 10 of A(1982) Combined shear-tension and shear compression strengths.

Eq.(A-4) can be written, with Y', Y, S as compression-, tension- and shear strengths:

$$\frac{\sigma_6}{S} = \sqrt{\frac{(1-\sigma_2/Y)\cdot(1+\sigma_2/Y')}{1+c\sigma_2/Y'}} \qquad \text{(A-5)}$$

With: $c = 3F_{266}Y'S^2$. (A-6)

When $c \to 1$, eq.(A-5) becomes: $\sigma_6 / S = \sqrt{1-\sigma_2/Y}$, or:

$$\left(\frac{\sigma_6}{S}\right)^2 + \left(\frac{\sigma_2}{Y}\right) = 1 \qquad \text{(A-7)}$$

which is the mixed mode Wu- equation of fracture mechanics, showing that micro-crack and macro crack extensions are the same. The exact derivation of this equation, in orthotropic stresses, is given in C(2011), Section 2.3, eq.(2.36):

$$1 = \frac{\sigma_2}{\xi_0\sigma_t/2} + \frac{\sigma_6^2}{\xi_0^2\sigma_t^2 n_6^2} = \frac{\sigma_2\sqrt{\pi c}}{\sigma_t\sqrt{\pi r_0/2}} + \frac{\left(\sigma_6\sqrt{\pi c}\right)^2}{\left(\sigma_t n_6\sqrt{2\pi r_0}\right)^2} = \frac{K_I}{K_{Ic}} + \frac{K_{II}^2}{K_{IIc}^2} \qquad \text{(A-8)}$$

because by the transformation from elliptical to circular coordinates: $\xi_0 = \sqrt{2r_0/c}$. Critical small crack propagation occurs at a critical crack density, when the crack distance is about the crack-length and is thus independent of the crack length, which can be chosen to have a standard value (depending on quality) and the second part of eq.(A-8) can be written as:

$$1=\frac{\sigma_2\sqrt{\pi c}}{\sigma_t\sqrt{\pi r_0/2}}+\frac{\left(\sigma_6\sqrt{\pi c}\right)^2}{\left(\sigma_t n_6\sqrt{2\pi r_0}\right)^2}=\frac{\sigma_2}{\sigma_{2c}}+\frac{\sigma_6^2}{\sigma_{6c}^2} \tag{A-9}$$

thus in deterministic ultimate strength values: σ_{2c},σ_{6c}.

The value of F_{266} in eq.(A-6), depends on the stability of the test, thus is not a constant, but a hardening factor, determining the amount of hardening at the, by the testing instability determined, ultimate state. This is shown e.g. by the following Fig. 4 of A(1993), where parameter values according to more stable torsion tube tests, are used to predict the oblique grain compression strength values. Because of more hardening, the peak of 1.1, at 10^0, is predicted, which can not be measured in the oblique grain test, due to earlier instability of the test setup.

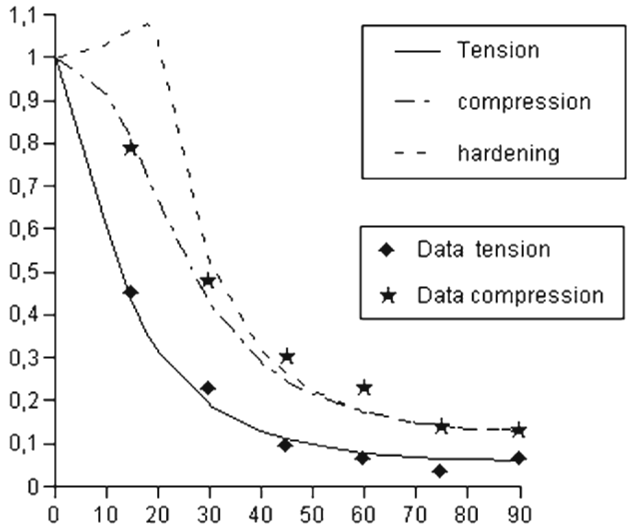
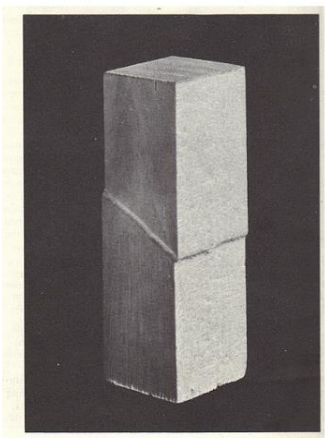
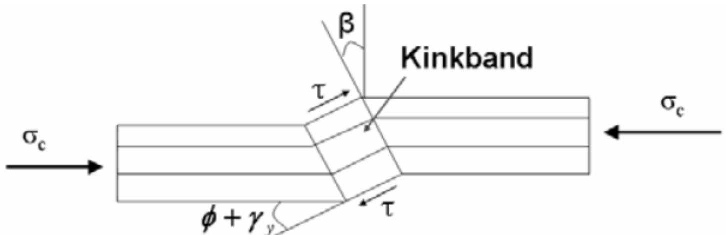


Fig. 4 of A(1993). Uniaxial oblique grain strength.

Determining for compression failure is the microscopic kinks formation in the cell walls, which is a buckling and plastic shearing mechanism. The kinks multiply and unite in kink-bands and kink-planes at fiber misalignments. Known by everyone is the slip-plane of the prism compression test showing a horizontal crease (shear line,





$$\sigma_c=\frac{K}{\phi+\gamma_y}$$

Fig. 1. Kinkband formation, where K is the plastic shear strength of the matrix (e.g. 11.3 Mpa), $\phi=15^0$ is the misalignment (e.g. for Spruce) γ_y is the longitudinal shear yield strain.

slip line) on the longitudinal radial plane, while on the longitudinal tangential plane the crease is inclined at 45^0 to 60^0 (depending on the species) with the vertical axis. For this bi-axial compression fracture, the same fracture mechanism occurs as for combined mode I-II fracture, discussed above. The shear loading of the micro-cracks is now due to misalignment component of the normal stress. Eq.(A-3) now becomes with $F_{12} = \sigma_6 = 0$:

$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + 3F_{112}\sigma_1^2\sigma_2 = 1$ or:

$$\sigma_1\left(\frac{1}{X} - \frac{1}{X'}\right) + \sigma_2\left(\frac{1}{Y} - \frac{1}{Y'}\right) + \left(\frac{\sigma_1^2}{XX'} + \frac{\sigma_2^2}{YY'}\right) + 3F_{112}\sigma_2\sigma_1^2 = 1 \qquad \text{(A-10)}$$

Above eq.(26) of A(2009), is eq.(A-10) closely approximated to eq.(A-11) below:

$$\frac{\sigma_1}{X'} = -\sqrt{\frac{(1-\sigma_2/Y)(1+\sigma_2/Y')}{1+c\sigma_2/Y'}} \approx -\sqrt{1-\sigma_2/Y} \qquad \text{(A-11)}$$

when the hardening constant $c = 3F_{112}Y'(X')^2$ approaches one: $c \approx 1$.

The parabolic Eq.(A-11) is shown in Fig. 5 of A(1993), given below, by the data points of the longitudinal compression side and is shown as fitted to this theoretical equation in fig. 6 of A(1993). As mentioned, this hardening of the torsion tube tests, is not found in the uniaxial oblique grain tests, which is earlier unstable, thus showing less hardening. Regarding the other third order hardening constants, it is to be expected that $F_{166} \approx 0$ for clear wood because σ_1 is in the same direction as the flat crack and thus not influenced by that crack. This also applies for F_{122}, which therefore also is zero and has no physical meaning and indeed is not present in fig. 5 below. Determining is F_{112}, representing hardening by kinking and slip-plane formation. According to fig.5 below is F_{112} zero at the longitudinal tension side.

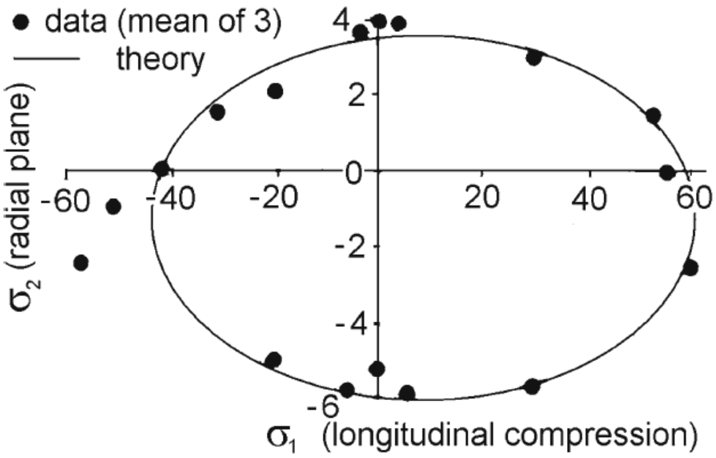


Fig. 5 of A(1993). Initial yield for $F_{12} = 0$ and $\sigma_6 = 0$

In A(1993) is shown that all data may show a different amount of hardening at failure. Therefore, meaningless higher order terms are necessary to give one equation to collect all data. Because tests in longitudinal compression show other and more hardening than tests in tension, separate data fits for longitudinal tension and longitudinal compression are necessary, as given by eq.(31) and eq.(32) of A(2009). For the parameter estimation by the uniaxial oblique grain tests, is in eq.(18) of A(2009):

$F_{12} = F_{122} = F_{166} = 0$; $3F_{112} \approx 0.9/((X')^2Y')$; $3F_{266} \approx 0.9/(S^2Y')$

Because hardening is not always guaranteed in real structures and test situations, the initial flow criterion for the Codes has to be:

$$\frac{\sigma_6^2}{S^2}+\frac{\sigma_1}{X}-\frac{\sigma_1}{X'}+\frac{\sigma_1^2}{XX'}+\frac{\sigma_2}{Y}-\frac{\sigma_2}{Y'}+\frac{\sigma_2^2}{YY'}=1 \qquad \text{(A-12)}$$

A.2. Some conclusions

It was for the first time shown in A(1982) that the tensor polynomial failure criterion applies to wood. Also is shown, that the fourth-degree and higher-degree polynomial terms have no physical meaning and thus are zero. Only the third-degree polynomial part is identical to the real failure criterion, while the third degree terms represent deviations from orthotropic behavior and represent post initial flow hardening behavior, which numerical value depends on the stability of the test specimen and testing device.
It also was for the first time shown that the second-degree part of the tensor polynomial is identical to the in A(1982) and A(2009) derived orthotropic extension of the isotropic critical distortional energy criterion for initial yield. The third degree polynomial hardening terms of the failure criterion are shown to represent the, in C(2011) theoretical derived, Wu-mixed-mode I-II fracture equation, showing hardening to be based on hindered micro-crack extension and micro-crack arrest. This also applies for slip plane formation of compression fracture, which is a variant of shear failure according to the mixed mode Wu-equation. Important is the conclusion that the failure criterion shows that micro-crack extension is always involved in fracture processes. The derivation of the new fracture mechanics theory, is therefore based on micro-crack extension. In C(2014) is the exact derivation given of the geometric correction factor for small crack extension towards the macro-crack tip. This correction factor appears to be numerical the same as for macro-crack extension.
For uniaxial loading, the failure criterion can be resolved in factors, leading to the derivation of extended Hankinson equations. This allows the relations between the constants of the total failure criterion to be elucidated as is necessary for data fitting of this criterion to provide a simple method to determine all strength parameters by simple uniaxial, oblique grain compression and tension tests. Based on this, the numerical failure criterion is given with the simple lower bound criterion for practice and for the codes in e.g. A(2009).
Because in limit analysis, the extremum variational principle applies for initial "flow" and thus the virtual work equations apply, the variations are sufficient small to get a linear irreversible process, and then the plastic potential function exists, which is identical to the yield function at flow, and for which the normality rule applies. This thus applies for the derived orthotropic critical distortional energy criterion, making complete exact solutions possible. The absence of coupling term, F_{12} =0, between the normal stresses means that the reinforcement takes only normal loading, causing the matrix to carry the whole shear loading. Failure of the matrix occurs before flow of the reinforcement. This follows e.g. from Balsa

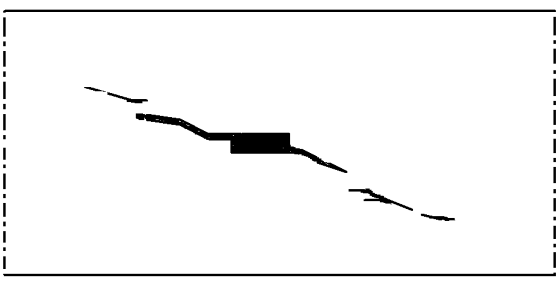


Fig. 1 of C(2011) - Shear failure by the asymmetric four point bending test with small center-slit. (Sketch after a photo)

wood, which is highly orthotropic, but shows the isotropic ratio of the critical stress intensities $K_{IIc} / K_{Ic} \approx 2$ of the isotropic matrix material at failure at initial flow. For dense, strong, (thus with a high reinforcement content) clear wood, this is shown by the oblique crack extension, according to Fig. 1 of C(2011), showing the isotropic oblique angle at the start of shear crack extension, and thus shows the matrix to be determining for initial failure. It is therefore a requirement for an exact orthotropic solution, applicable to wood, to satisfy the equilibrium condition for the total orthotropic stresses, as well as for the isotropic solution of the stresses in the matrix at failure. This last condition is not satisfied in all other existing fracture mechanics models.
Early failure of the matrix causes stress redistribution of mainly shear with compression in the matrix and increased tensile stress in the fibres. The measured negative contraction for creep in tension indicates this mechanism. As in reinforced concrete, truss action is possible, as noticeable by the strong negative contraction coefficient (swelling instead of contraction) in the bending tensile zone of the beam. Failure in compression is determined by the difference in the principal compression stresses. Thus the maximal shear stress or Tresca criterion applies. The necessary validity of the Tresca criterion is confirmed by D(2008b) and D(2008a), where the strongly increased (sixfold) compression strength under the load of locally loaded blocks and the increased embedding strength of dowels is explained by the construction of the equivalent slip line field in the specimen based on the Tresca criterion. In addition, the many apparent contradictions of the different investigations are explained by this theory. This strong increase of the compression strength is due to confined dilatation by real hardening (when the empty spaces in wood are pressed away)..
The existence of an isotropic matrix in wood (lignin with branched hemicellulose) follows not only from material analysis, but also, as mentioned, from the high compression strength at confined dilation with the absence of failure by triaxial hydrostatic compression, (what is not the case for orthotropy, because then, for equal triaxial stresses, the strains then are not equal and yield remains possible).
Plastic flow in wood starts with propagation of empty spaces by segmental jumps, just as the dislocation propagation in steel and the possibility should be accounted that there is no change in density at initial flow (as for steel) and the plastic incompressibility condition should be accounted as possibility, and as follows from the normality rule of flow in combination with perfect plasticity, the Tresca criterion (maximal shear stress criterion) then also should apply. The Tresca criterion should be the inscribed polygon within the von Mises criterion and thus provides a lower bound calculation of the strength of the von Mises wood material. By the dissipation according to the incompressibility condition, the minimum energy principle is followed providing the lowest possible upper bound and therefore the closest to the exact flow criterion.
Limit analysis therefore has to be based on incompressibility and the Tresca criterion.
It has to be stressed for the virtual work equations of limit analysis that neither the chosen equilibrium, nor the compatible strain and displacement set need not be the actual state, nor need the equilibrium and compatible sets to be related in any way to each other.

A.3. References:

The author name “van der Put T.A.C.M.”, (shortcut “vdPut”) is left out in the following:

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D(2008b) Explanation of the embedding strength of particle board, Holz Roh Werkst 66: 259–265
D(2008a) Derivation of the bearing strength perpendicular to the grain of locally loaded timber blocks, Holz Roh Werkst 66: 409–417
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